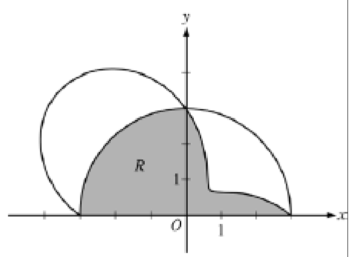
BC Calculus
Ch. 10 Review

Name: ______________________________
Period: _____

Find the area of the region for (1) without using a calculator. On 2-4 you may use a calculator to find the area, but clearly show the integral used to find the area.

1. The area enclosed by $r=4\cos(5\theta)$	2. Common interior of $r=\sqrt{2}$ and $r=2\cos(\theta)$.	3. Common interior of $r=3-2\sin(\theta)$ and $r=-3+2\sin(\theta)$	4. Inside $r=3\sin(\theta)$ and outside $r=2-\sin\theta$

5. The graphs of the polar curves $r=3$ and $r=3-2\sin(2\theta)$ are shown in the figure above for $0 \le \theta \le \pi$.



(a) Let R be the shaded region that is inside the graph of $r=3$ and inside the graph of $r=3-2\sin(2\theta)$. Find the area of R.

(b) For the curve $r=3-2\sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at $\theta=\frac{\pi}{6}$.

(c) The distance between the two curves changes for $0<\theta<\frac{\pi}{2}$.

Find the rate at which the distance between the two curves is changing with respect to θ when $\theta=\frac{\pi}{3}$.

(d) A particle is moving along the curve $r=3-2\sin(2\theta)$ so that $\frac{d\theta}{dt}=3$ for all times $t \ge 0$. Find the value of $\frac{dr}{dt}$ at $\theta=\frac{\pi}{6}$.

6. A population is modeled by a function P that satisfies the logistic differential equation

$$\frac{dP}{dt}=\frac{P}{5}\left(1-\frac{P}{12}\right).$$

(a) If $P(0)=3$, what is $\lim_{t\to\infty} P(t)$?

If $P(0)=20$, what is $\lim_{t\to\infty} P(t)$?

(b) If $P(0)=3$, for what value of P is the population growing the fastest?

(c) A different population is modeled by a function Y that satisfies the separable differential equation

$$\frac{dY}{dt}=\frac{Y}{5}\left(1-\frac{t}{12}\right).$$

Find $Y(t)$ if $Y(0)=3$.

(d) For the function Y found in part (c), what is $\lim_{t\to\infty} Y(t)$?

7. At time t, a particle moving in the xy-plane is at position $(x(t), y(t))$, where $x(t)$ and $y(t)$ are not explicitly given. For $t \ge 0$, $\frac{dx}{dt}=4t+1$ and $\frac{dy}{dt}=\sin(t^2)$. At time $t=0$, $x(0)=0$ and $y(0)=-4$.

(a) Find the speed of the particle at time $t=3$, and find the acceleration vector of the particle at time $t=3$.

(b) Find the slope of the line tangent to the path of the particle at time $t=3$.

(c) Find the position of the particle at time $t=3$.

(d) Find the total distance traveled by the particle over the time interval $0 \le t \le 3$.

8. If $x=t^2+1$ and $y=t^3$, then $\frac{d^2y}{dx^2}=$

(A) $\frac{3}{4t}$ (B) $\frac{3}{2t}$ (C) $3t$ (D) $6t$ (E) $\frac{3}{2}$

9. For any time $t \ge 0$, if the position of a particle in the xy-plane is given by $x=t^2+1$ and $y=\ln(2t+3)$, then the acceleration vector is

(A) $\left(2t, \frac{2}{(2t+3)}\right)$ (B) $\left(2t, \frac{-4}{(2t+3)^2}\right)$ (C) $\left(2, \frac{4}{(2t+3)^2}\right)$

(D) $\left(2, \frac{2}{(2t+3)^2}\right)$ (E) $\left(2, \frac{-4}{(2t+3)^2}\right)$

10. Consider the curve in the xy-plane represented by $x=e^t$ and $y=te^{-t}$ for $t \ge 0$. The slope of the line tangent to the curve at the point where $x=3$ is

(A) 20.086 (B) 0.342 (C) –0.005 (D) –0.011 (E) –0.033

11. If a particle moves in the xy-plane so that at time $t>0$ its position vector is $\left(\ln(t^2+2t), 2t^2\right)$, then at time $t=2$, its velocity vector is

(A) $\left(\frac{3}{4}, 8\right)$ (B) $\left(\frac{3}{4}, 4\right)$ (C) $\left(\frac{1}{8}, 8\right)$ (D) $\left(\frac{1}{8}, 4\right)$ (E) $\left(-\frac{5}{16}, 4\right)$

12. Which of the following is equal to the area of the region inside the polar curve $r=2\cos\theta$ and outside the polar curve $r=\cos\theta$?

(A) $3\int_0^{\frac{\pi}{2}}\cos^2\theta\, d\theta$ (B) $3\int_0^{\pi}\cos^2\theta\, d\theta$ (C) $\frac{3}{2}\int_0^{\frac{\pi}{2}}\cos^2\theta\, d\theta$ (D) $3\int_0^{\frac{\pi}{2}}\cos\theta\, d\theta$ (E) $3\int_0^{\pi}\cos\theta\, d\theta$

13. The length of the path described by the parametric equations $x=\frac{1}{3}t^3$ and $y=\frac{1}{2}t^2$, where $0 \le t \le 1$, is given by

(A) $\int_0^1\sqrt{t^2+1}\,dt$

(B) $\int_0^1\sqrt{t^2+t}\,dt$

(C) $\int_0^1\sqrt{t^4+t^2}\,dt$

(D) $\frac{1}{2}\int_0^1\sqrt{4+t^4}\,dt$

(E) $\frac{1}{6}\int_0^1 t^2\sqrt{4t^2+9}\,dt$

14. If f is a vector-valued function defined by $f(t)=\left(e^{-t}, \cos t\right)$, then $f''(t)=$

(A) $-e^{-t}+\sin t$ (B) $e^{-t}-\cos t$ (C) $\left(-e^{-t}, -\sin t\right)$

(D) $\left(e^{-t}, \cos t\right)$ (E) $\left(e^{-t}, -\cos t\right)$

15. A population of a certain region follows the logistic growth model

$$\frac{dp}{dt}=0.02p-0.004p^2, \quad p(0)=2 \text{ million}.$$

What is the environmental carrying capacity of this region?

(a) 15 million **(b)** 10 million **(c)** 3 million **(d)** 0.2 million **(e)** 5 million