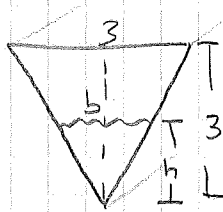


Z-#4: 26a, 28, 31ab, 35ab, 36, 43, & 329: 8, 9, 25, 30, 41, 48, 51, 56, 57, 59, 63, 64, 71ab, 77, 78

|            |                   |
|------------|-------------------|
| 26a) $h=1$ | $\frac{dh}{dt}=?$ |
| $b=1$      | $\frac{db}{dt}$   |
| $V$        | $\frac{dV}{dt}=2$ |



$$\frac{3}{3} = \frac{b}{h}$$

$$3b = 3h$$

$$b = h$$

$$V = \frac{1}{2}bh \cdot 12$$

$$V = 6bh$$

$$V = 6h^2$$

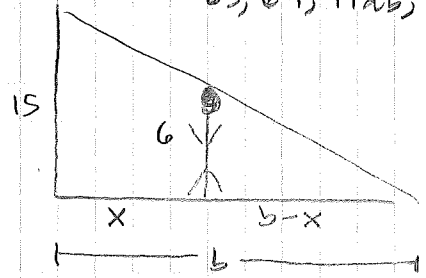
$$\frac{dV}{dt} = 12h \frac{dh}{dt}$$

$$2 = 12(1) \frac{dh}{dt}$$

$$\frac{1}{6} = \frac{dh}{dt}$$

$\frac{1}{6} \text{ ft/min}$

35.



a)  $\frac{dx}{dt} = 5$  Find  $\frac{db}{dt}$  when  $x=10$

$$\frac{15}{b} = \frac{6}{b-x}$$

$$6b = 15b - 15x$$

$$-9b = -15x$$

$$b = \frac{5}{3}x$$

$$\frac{db}{dt} = \frac{5}{3} \frac{dx}{dt}$$

b)  $s = b - x$

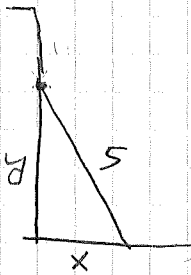
$$\frac{ds}{dt} = \frac{db}{dt} - \frac{dx}{dt}$$

$$= \frac{25}{3} - 5$$

$$10\frac{1}{3} \text{ ft/sec}$$

$$\frac{db}{dt} = \frac{5}{3}(5) = \frac{25}{3} \text{ ft/sec}$$

28.



Find  $\frac{dx}{dt}$  when  $x = \frac{2.5}{2}$   
 $y = \frac{5\sqrt{3}}{2}$

$$x^2 + y^2 = 5^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

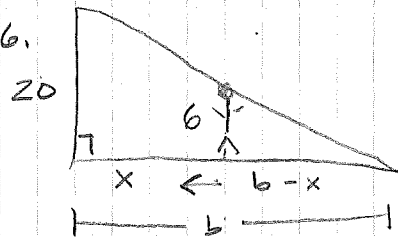
$$\frac{dx}{dt} = -\frac{y}{x} \frac{dy}{dt}$$

$$= -\frac{5\sqrt{3}}{2} \cdot \frac{2}{5} (-.1)$$

$$= \frac{15\sqrt{3}}{3\sqrt{3}} = \frac{5}{1} = 5$$

$\frac{dy}{dt} = .260 \text{ m/s}$

36.



$$\frac{b}{20} = \frac{b-x}{6}$$

$$20b - 20x = 6b$$

$$14b = 20x$$

$$7b = 10x$$

$$7 \frac{db}{dt} = 10 \frac{dx}{dt}$$

$$\frac{db}{dt} = \frac{10}{7}(5) = \frac{50}{7} \text{ ft/s}$$

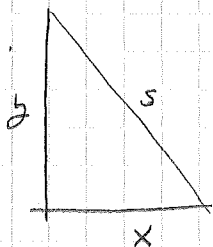
b)  $s = b - x$

$$\frac{ds}{dt} = \frac{db}{dt} - \frac{dx}{dt}$$

$$= \frac{50}{7} - \frac{5}{1} = \frac{45}{7}$$

$$= \frac{15}{7}$$

31.



$$x^2 + y^2 = 5^2$$

$$2x \frac{dx}{dt} + \frac{dy}{dt} \cdot 2y = 25 \frac{ds}{dt}$$

$$\frac{x \frac{dx}{dt} + y \frac{dy}{dt}}{s} = \frac{ds}{dt}$$

|           |                        |
|-----------|------------------------|
| $x = 150$ | $\frac{dx}{dt} = -450$ |
|-----------|------------------------|

|           |                        |
|-----------|------------------------|
| $y = 200$ | $\frac{dy}{dt} = -600$ |
|-----------|------------------------|

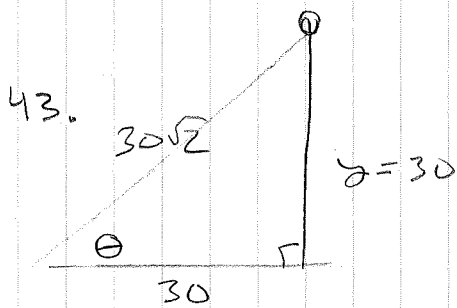
|           |                   |
|-----------|-------------------|
| $s = 250$ | $\frac{ds}{dt} =$ |
|-----------|-------------------|

$$\frac{(150)(-450) + (200)(-600)}{250} = \frac{ds}{dt}$$

$$-750 \text{ mi/hr} = \frac{ds}{dt}$$

a) 750 mi/hr

b)  $t = \frac{D}{r} = \frac{250 \text{ mi}}{750 \text{ mi/hr}} = .33 \text{ hr} = 20 \text{ min}$



Find  $\frac{d\theta}{dt}$  when

$$\frac{dy}{dt} = 3 \text{ m/s when } y = 30$$

$$\tan \theta = y/30$$

$$\sec^2 \theta \frac{d\theta}{dt} = \frac{1}{30} \frac{dy}{dt}$$

$$\frac{d\theta}{dt} = \cos^2 \theta \cdot \frac{1}{30} (3)$$

$$= \left(\frac{30}{30\sqrt{2}}\right)^2 \cdot \frac{1}{30}$$

$$= \frac{1}{2} \cdot \frac{1}{10} = \frac{1}{20} \frac{\text{radians}}{\text{sec}}$$

P. 329.

8. d

9. a

25.  $3 \ln(x^2 - 1) - 9 \ln(x)$

30.  $3 \ln(x) + 2 \ln(y) - 4 \ln(z)$

$$\ln(x^3) + \ln(y^2) - \ln(z^4)$$

$$\ln\left(\frac{x^3 y^2}{z^4}\right)$$

41.  $y = \ln(x^3)$

$$y = 3 \ln(x)$$

$$y' = \frac{3}{x}$$

$$y'(1) = 3$$

$$y - 0 = 3(x - 1)$$

$$y = 3x - 3$$

48.  $y = x \ln(x)$

$$y' = x \cdot \frac{1}{x} + \ln(x)$$

$$y' = 1 + \ln(x)$$

51.  $f(x) = \ln\left(\frac{x}{x^2+1}\right)$

$$f'(x) = \frac{(x^2+1) \cdot (x^2+1)(1) - (x)(2x)}{x(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2}{x(x^2+1)}$$

$$= \frac{1-x^2}{x(x^2+1)}$$

51.  $f(x) = \ln\left(\frac{x}{x^2+1}\right) = \ln(x) - \ln(x^2+1)$

$$f'(x) = \frac{1}{x} - \frac{2x}{x^2+1}$$

56.  $y = \ln(\ln(x))$

$$y' = \frac{1}{\ln(x)} \cdot \frac{1}{x}$$

$$y' = \frac{1}{x \ln(x)}$$

57.  $y = \ln\sqrt{\frac{x+1}{x-1}} = \frac{1}{2} \ln(x+1) - \frac{1}{2} \ln(x-1)$

$$y' = \frac{1}{2(x+1)} - \frac{1}{2(x-1)}$$

$$\frac{x-1-(x+1)}{2(x+1)(x-1)} = \frac{-2}{2(x^2-1)} = \boxed{\frac{-1}{x^2-1}}$$

59.  $f(x) = \ln\left(\frac{\sqrt{4+x^2}}{x}\right)$

$$= \ln(\sqrt{4+x^2}) - \ln(x)$$

$$= \frac{1}{2} \ln(4+x^2) - \ln(x)$$

$$f'(x) = \frac{2x}{2(4+x^2)} - \frac{1}{x}$$

$$= \frac{x}{4+x^2} - \frac{1}{x}$$

$$63. \quad y = \ln|\sin x|$$

$$y' = \frac{1}{\sin x} \cdot \cos x$$

$$y' = \cot x$$

$$64. \quad y = \ln|\csc x|$$

$$y' = \frac{1}{\csc x} \cdot -\csc x \cot x$$

$$y' = -\cot x$$

$$71a) \quad f(x) = 3x^2 - \ln x$$

$$f'(x) = 6x - \frac{1}{x}$$

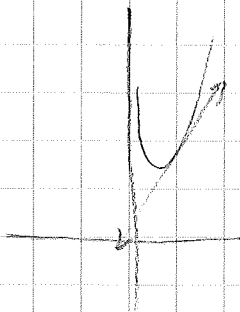
$$f'(1) = 6(1) - \frac{1}{1}$$

$$= 6 - 1$$

$$= 5$$

$$y - 3 = 5(x - 1)$$

b)



$$77. \quad x^2 - 3\ln(y) + y^2 = 10$$

$$2x - 3\left(\frac{1}{y}\right)y' + 2yy' = 0$$

$$-\frac{3}{y}y' + 2yy' = -2x$$

$$y'(-\frac{3}{y} + 2y) = -2x$$

$$y' = \frac{-2x}{2y - \frac{3}{y}}$$

$$y' = \frac{-2xy}{2y^2 - 3} = \frac{2xy}{3 - 2y^2}$$

2-4

cont'd

$$78. \quad \ln(xy) + 5x = 30$$

$$\frac{1}{xy} \cdot (xy' + y(1)) + 5 = 0$$

$$\frac{1}{xy} (xy' + y) = -5$$

$$xy' + y = -5xy$$

$$xy' = -5xy - y$$

$$y' = \frac{-5xy - y}{x}$$

$$y' = -\frac{5xy + y}{x}$$

or  $y' = -5y - \frac{y}{x}$

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