

8-#1: 1-5, 7, 9, 11, 13, 19, 21, 23, 28, 30, 31, 35, 36, 37, 41, 47, 53b

1. $\frac{dy}{dx} = \frac{x}{\sqrt{x^2+1}}$

$\int \frac{x}{\sqrt{x^2+1}} dx$ $u = x^2+1$
 $\frac{1}{2} \int u^{-1/2} du$ $du = 2x dx$
 $\frac{1}{2} \cdot 2 u^{1/2} + C$
 $\sqrt{x^2+1} + C$ [B]

2. $\int \frac{x}{x^2+1} dx$ $u = x^2+1$
 $\frac{1}{2} \int \frac{1}{u} du$ $du = 2x dx$
 $\frac{1}{2} \ln|u| + C$ $\frac{1}{2} du = x dx$
 $\frac{1}{2} \ln(x^2+1) + C$
 $\ln(\sqrt{x^2+1}) + C$ [A]

3. $\int \frac{1}{x^2+1} dx = \tan^{-1}(x) + C$ [C]

4. $\int x \cos(x^2+1) dx$ $u = x^2+1$
 $\frac{1}{2} \int \cos u du$ $du = 2x dx$
 $\frac{1}{2} \sin(u(x^2+1)) + C$ $\frac{1}{2} du = x dx$
 [C]

5. $\int (3x-2)^4 dx$ $u = 3x-2$
 $\frac{1}{3} \int u^4 du$ $du = 3 dx$
 $\frac{1}{3} \cdot \frac{1}{5} u^5 + C$ $\frac{1}{3} du = dx$
 $\frac{1}{15} (3x-2)^5 + C$

7. $\int \frac{1}{\sqrt{x}(1-2\sqrt{x})} dx$ $u = 1-2\sqrt{x}$
 $du = -2 \cdot \frac{1}{2} x^{-1/2} dx$
 $-du = \frac{1}{\sqrt{x}} dx$
 $-\int \frac{1}{u} du$
 $-\ln|u| + C$
 $-\ln|1-2\sqrt{x}| + C$

9. $\int \frac{3}{\sqrt{1-t^2}} dt$
 $3 \sin^{-1}(t) + C$

11. $\int t \sin(t^2) dt$ $u = t^2$
 $\frac{1}{2} \int \sin(u) du$ $du = 2t dt$
 $-\frac{1}{2} \cos(t^2) + C$ $\frac{1}{2} du = t dt$

13. $\int \cos x e^{\sin x} dx$ $u = \sin x$
 $\int e^u du$ $du = \cos x dx$
 $e^u + C$
 $e^{\sin x} + C$

19. $\int v + \frac{1}{(3v-1)^3} dv$ $u = 3v-1$
 $\frac{1}{2} v^2 + \frac{1}{3} \int u^{-3} du$ $du = 3 dv$
 $\frac{1}{2} v^2 + \frac{1}{3} \cdot \frac{1}{-2} u^{-2} + C$ $\frac{1}{3} du = dv$
 $\frac{1}{2} v^2 - \frac{1}{6} (3v-1)^{-2} + C$

21. $\int \frac{t^2-3}{-t^3+9t+1} dt$ $u = -t^3+9t+1$
 $du = (-3t^2+9t) dt$
 $-\frac{1}{3} \int \frac{1}{u} du$ $du = -3(t^2-3) dt$
 $-\frac{1}{3} \ln|-t^3+9t+1| + C$

23. $\int \frac{x^2}{x-1} dx$ ← improper
 $\int x+1 + \frac{1}{x-1} dx$
 $\frac{1}{2} x^2 + x + \ln|x-1| + C$

$$\frac{x-1 \sqrt{x^2-x+1} - (x^2-x)}{-(x-1)}$$

$$\frac{x}{-(x-1)}$$

$$\frac{1}{1}$$

28. $\int x \left(1 + \frac{1}{x}\right)^3 dx$
 $x \left(1 + \frac{1}{x}\right) \left(1 + \frac{1}{x}\right) \left(1 + \frac{1}{x}\right)$
 $x \left(1 + \frac{2}{x} + \frac{1}{x^2}\right) \left(1 + \frac{1}{x}\right)$
 $x \left(1 + \frac{3}{x} + \frac{3}{x^2} + \frac{1}{x^3}\right)$
 $x + 3 + 3x^{-1} + x^{-2}$
 $\int x + 3 + 3x^{-1} + x^{-2} dx$
 $\frac{1}{2} x^2 + 3x + 3 \ln|x| - \frac{1}{x} + C$

30. $\int \sec(4x) dx$
 $\frac{1}{4} \ln|\sec(4x) + \tan(4x)| + C$

31. $\int \csc(\pi x) \cot(\pi x) dx$
 $-\frac{1}{\pi} \csc(\pi x) + C$

$$35. \int \frac{2}{e^{-x}+1} dx \quad \text{multiply top and bottom by } e^x.$$

$$\int \frac{2e^x}{e^0+e^x} dx$$

$$\int \frac{2e^x}{1+e^x} dx \quad u=1+e^x$$

$$2 \int \frac{1}{u} du \quad du=e^x dx$$

$$2 \ln|1+e^x| + C$$

$$36. \int \frac{5}{3e^x-2} dx \cdot \frac{e^{-x}}{e^{-x}}$$

$$\int \frac{5e^{-x}}{3e^0-2e^{-x}} dx$$

$$\int \frac{5e^{-x}}{3-2e^{-x}} dx \quad u=3-2e^{-x}$$

$$\frac{5}{2} \int \frac{1}{u} du \quad du=2e^{-x} dx$$

$$\frac{5}{2} \ln|3-2e^{-x}| + C$$

$$37. \int \frac{\ln(x^2)}{x} dx$$

$$\int \frac{2 \ln(x)}{x} dx \quad u=\ln(x)$$

$$2 \int u du \quad du=1/x dx$$

$$2 \cdot \frac{1}{2} u^2 + C$$

$$(\ln(x))^2 + C$$

$$\ln^2(x) + C$$

$$41. \int \frac{1}{\cos \theta - 1} d\theta \quad \frac{\cos \theta + 1}{\cos \theta + 1}$$

$$\int \frac{\cos \theta + 1}{\cos^2 \theta - 1} d\theta$$

$$\int \frac{\cos \theta + 1}{-(1 - \cos^2 \theta)} d\theta$$

$$- \int \frac{\cos \theta + 1}{\sin^2 \theta} d\theta$$

$$- \int \frac{\cos \theta}{\sin \theta} \cdot \frac{1}{\sin \theta} + \csc^2 \theta d\theta$$

continued \uparrow

$$41. - \int \cot \theta \csc \theta + \csc^2 \theta d\theta$$

$$-(\csc \theta - \cot \theta) + C$$

$$\csc \theta + \cot \theta + C$$

$$53b. \int (\sec \theta + \tan \theta)^2 d\theta$$

$$\int \sec^2 \theta + 2 \sec \theta \tan \theta + \tan^2 \theta d\theta$$

$$\int \sec^2 \theta + 2 \sec \theta \tan \theta + (\sec^2 \theta - 1) d\theta$$

$$\int 2 \sec^2 \theta + 2 \sec \theta \tan \theta - 1$$

$$2 \tan \theta + 2 \sec \theta + C$$

$$67. \int_0^{2/\sqrt{3}} \frac{1}{4+9x^2} dx$$

$$\frac{1}{4} \int_0^{2/\sqrt{3}} \frac{1}{1+(3/2x)^2} dx \quad u=3/2x$$

$$\frac{1}{4} \cdot \frac{2}{3} \int_0^{\sqrt{3}} \frac{1}{1+u^2} du \quad du=3/2 dx$$

$$\frac{1}{6} \left[\tan^{-1} u \right]_0^{\sqrt{3}} \quad 2/3 du = dx$$

$$\frac{1}{6} \left[\frac{\pi}{3} - 0 \right] = \frac{\pi}{18} \quad u(2/\sqrt{3}) = \frac{3}{2} \cdot \frac{2}{\sqrt{3}}$$

$$\frac{3\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{3\sqrt{3}}{3}$$

p. 559: 9, 11, 13, 15

$$9. \int \frac{3}{x^2+x-2} dx$$

$$\frac{3}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$$

$$\int \frac{-1}{x+2} + \frac{1}{x-1} dx$$

$$x=1 \quad 3 = A(x-1) + B(x+2)$$

$$3 = 3B$$

$$1 = B$$

$$x=-2 \quad 3 = -3A$$

$$-1 = A$$

$$-\ln|x+2| + \ln|x-1| + C$$

$$11. \int \frac{5-x}{2x^2+x-1} dx$$

$$\frac{5-x}{(2x-1)(x+1)} = \frac{A}{2x-1} + \frac{B}{x+1}$$

$$\int \frac{3}{2x-1} + \frac{-2}{x+1} dx$$

$$5-x = A(x+1) + B(2x-1)$$

$$x=-1 \quad 6 = B(-3)$$

$$-2 = B$$

$$\frac{3}{2} \ln|2x-1| - 2 \ln|x+1| + C \quad x=1/2$$

$$4 \cdot 1/2 = A = 3/2$$

$$1/2 = 3/2 A$$

$$2/3 \cdot 1/2 = A$$

$$3 = A$$

$$13. \int \frac{x^2+12x+12}{x(x^2-4)} dx$$

$$\frac{x^2+12x+12}{x(x+2)(x-2)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-2}$$

$$\int \frac{x^2+12x+12}{x(x+2)(x-2)} dx$$

$$x^2+12x+12 = A(x+2)(x-2) + B(x)(x-2) + C(x)(x+2)$$

$$x=2 \quad 4+24+12 = 0+0+C(2)(4)$$

$$40 = 8C$$

$$5 = C$$

$$x=-2 \quad 4-24+12 = 0+B(-2)(-4)+0$$

$$-8 = 4B$$

$$-2 = B$$

$$x=0 \quad 12 = -4A+0+0$$

$$-3 = A$$

$$\int \left[\frac{-3}{x} + \frac{-2}{x+2} + \frac{5}{x-2} \right] dx$$

$$-3 \ln|x| - \ln|x+2| + 5 \ln|x-2| + C$$