8-#3: p. 531: 13, 26, 29, 35, 58, 73-78, p. 559: 21, 27, 29, 53, 55ab

13. $\int x^3 e^x \, dx$

| | | |
|---|---|---|
| + | $x^3$ | $e^x$ |
| − | $3x^2$ | $e^x$ |
| + | $6x$ | $e^x$ |
| − | $6$ | $e^x$ |
| + | $0$ | $e^x$ |

$x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$

26. $\int \frac{x}{\sqrt{2+3x}} \, dx$

$u = 2+3x$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$3x = u-2$

$x = \frac{1}{3}(u-2)$

$\int u^{-1/2} \cdot \frac{1}{3}(u-2) \cdot \frac{1}{3} \, du$

$\frac{1}{9}\int u^{1/2} - 2u^{-1/2} \, du$

$\frac{1}{9}\left[\frac{2}{3}u^{3/2} - 2 \cdot 2u^{1/2}\right]$

$\frac{2}{27}(2+3x)^{3/2} - \frac{4}{9}(2+3x)^{1/2} + C$

Factor:

$\frac{2}{9}(2+3x)^{1/2}\left[\frac{1}{3}(2+3x) + 2\right] + C$

$\frac{2}{9}(2+3x)^{1/2}\left[x + 4/3\right] + C$

$\frac{2}{27}(2+3x)^{1/2}[3x+4] + C$

29. $\int x^3 \sin x \, dx$

| | | |
|---|---|---|
| + | $x^3$ | $\sin x$ |
| − | $3x^2$ | $-\cos x$ |
| + | $6x$ | $-\sin x$ |
| − | $6$ | $\cos x$ |
| + | $0$ | $\sin x$ |

$-x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x + C$

35. $\int e^{2x} \sin x \, dx$

$u = \sin(x)$ $\quad v = \frac{1}{2}e^{2x}$

$du = \cos x \, dx$ $\quad dv = e^{2x} \, dx$

$\sin(x) \cdot \frac{1}{2}e^{2x} - \frac{1}{2}\int e^{2x}\cos x \, dx$

$u = \cos x$ $\quad v = \frac{1}{2}e^{2x}$

$du = -\sin x \, dx$ $\quad dv = e^{2x} \, dx$

$\int e^{2x}\sin x \, dx = \frac{1}{2}e^{2x}\sin(x) - \frac{1}{2}\left[\cos x \cdot \frac{1}{2}e^{2x} + \frac{1}{2}\int \sin x \, e^{2x} \, dx\right]$

$\int e^{2x}\sin x \, dx = \frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^x\cos x - \frac{1}{4}\int \sin x \, e^{2x} \, dx$

$5/4 \int e^{2x}\sin x \, dx = \frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^x\cos x$

$\int e^{2x}\sin x \, dx = \frac{2}{5}e^{2x}\sin x - \frac{1}{5}e^x\cos x + C$

* 35. $\int e^{2x} \sin x\,dx$ $u = e^{2x}$ $v = -\cos x$

$du = 2e^x\,dx$ $dv = \sin x\,dx$

again with $u = e^{2x}$

$-e^{2x}\cos x + 2\int e^{2x}\cos x\,dx$

$u = e^{2x}$ $u = \sin x$

$du = 2e^{2x}$ $dv = \cos x\,dx$

$\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2\left[e^{2x}\sin x - 2\int e^{2x}\sin x\,dx\right]$

$\int e^{2x}\sin x\,dx = -e^{2x}\cos x + 2e^{2x} - 4\int e^{2x}\sin x\,dx$

$5\int e^{2x}\sin x\,dx = e^{-2x}\cos x + 2e^{2x}$

$\int e^{2x}\sin x\,dx = \frac{1}{5}e^{-2x}\cos(x) + 2e^{2x} + C$

58. $\int_0^{\pi/4} x\sec^2 x\,dx$ $u = x$ $v = \tan x$

$du = dx$ $dv = \sec^2 x\,dx$

$[x\tan x]_0^{\pi/4} - \int_0^{\pi/4}\tan x\,dx$

$\left[x\tan x + \ln|\cos x|\right]_0^{\pi/4}$

$\frac{\pi}{4}\tan\frac{\pi}{4} + \ln\left|\cos\left(\frac{\pi}{4}\right)\right| - [0 + \ln|\cos(0)|]$

$\pi/4(1) + \ln(\sqrt{2}/2) - 0 + 0 = \boxed{\frac{\pi}{4} + \ln\left(\frac{\sqrt{2}}{2}\right)}$

73. u-sub $u = \ln x$ $du = \frac{1}{x}dx$

74. parts $u = \ln(x)$ $dv = x\,dx$

75. parts $u = x^2$ $dv = e^{2x}dx$

76. u-sub $u = x^2$ $du = 2x\,dx$

* 77. u-sub $u = x+1$ $du = dx$ $x = u-1$ could do parts

78. u-sub $u = x^2+1$ $du = 2x\,dx$

p. 559:

21. $\int \frac{x^2-1}{x^3+x}\,dx$

$$\int \frac{-1}{x}\,dx + \int \frac{2x}{x^2+1}\,dx = -\ln|x| + \ln(x^2+1) + C$$

$$\frac{x^2-1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

$x^2-1 = A(x^2+1) + (Bx+C)(x)$

$x=0:\ -1 = A(0^2+1) + 0$

$\boxed{-1 = A}$

$x^2-1 = (-1)(x^2+1) + Bx^2 + Cx$

$x^2-1 = -x^2-1 + Bx^2 + Cx$

$x^2+0x-1 = (B-1)x^2 + Cx - 1$

$B-1 = 1$

$\boxed{B=2}$

$\boxed{C=0}$

27. $\int \frac{x^2+5}{x^3-x^2+x+3}\,dx$

$-1$ | 1 −1 1 3

↓ −1 2 −3

1 −2 3 0

$$\int \frac{x^2+5}{(x+1)(x^2-2x+3)}\,dx$$

$$\int \frac{1}{x+1} + \frac{2}{x^2-2x+3}\,dx = \int \frac{1}{x+1} + \frac{2}{(x-1)^2+2}\,dx =$$

$u = \frac{1}{\sqrt{2}}(x-1)$

$du = \frac{1}{\sqrt{2}}\,dx$

$\sqrt{2}\,du = dx$

$$\int \frac{1}{x+1}\,dx + \frac{1}{2}\int \frac{2\,dx}{\left(\frac{x-1}{\sqrt{2}}\right)^2+1}$$

$$\ln|x+1| + \sqrt{2}\int \frac{1}{u^2+1}\,du$$

$$\ln|x+1| + \sqrt{2}\tan^{-1}\left(\frac{x-1}{\sqrt{2}}\right) + C$$

ouch!

$$\frac{x^2+5}{(x+1)(x^2-2x+3)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-2x+3}$$

$x^2+5 = A(x^2-2x+3) + (Bx+C)(x+1)$

$x=-1:\ 6 = A(1+2+3) + 0$

$6 = A(6)$

$\boxed{A=1}$

$x^2+5 = 1(x^2-2x+3) + Bx^2 + Bx + Cx + C$

$x^2+5 = (B+1)x^2 + (B+C-2)x + C+3$

$B+1 = 1$ $\quad$ $C+3 = 5$

$\boxed{B=0}$ $\quad$ $\boxed{C=2}$

29. $\int_0^1 \frac{3}{2x^2+5x+2}\,dx$

$$\int_0^1 \frac{2}{2x+1} + \frac{-1}{x+2}\,dx = \left[\frac{2}{2}\ln|2x+1| - \ln|x+2|\right]_0^1$$

$$\frac{3}{(2x+1)(x+2)} = \frac{A}{2x+1} + \frac{B}{x+2}$$

$3 = A(x+2) + B(2x+1)$

$x=-2$: $3 = A(0) + B(-4+1)$

$3 = -3B$

$-1 = B$

$x=-1/2$: $3 = A(-1/2+4/2) + 0$

$3 = 3/2\,A$

$6 = 3A$

$A = 2$

$\ln(3) - \ln(3) - [\ln(1) - \ln(2)]$

$0 - 0 + \ln(2)$

$\boxed{\ln(2)}$

53. long division, $\frac{x^3}{x-5}$ is improper

55a) u-sub $u = x^2+2x-8$

$du = 2x+2\,dx$

$du = 2(x+1)\,dx$

b) partial fractions