9-#10: p. 666: 1, 3, 7, 11, 13, 15, 17, 25, 79, 80.
p. 688: 23-27.

1. $\sum_{n=0}^{\infty} n x^n$

Ratio Test

$$\lim_{n\to\infty}\left|\frac{(n+1)x^{n+1}}{n\,x^n}\right|$$

$$\lim_{n\to\infty}\left|\frac{(n+1)\cancel{x^n}\cdot x^1}{n\,\cancel{x^n}}\right|$$

$$\lim_{n\to\infty}\frac{n+1}{n}\cdot|x| = |x|$$

Convergent if
$|x|<1$
$-1<x<1$

Inconclusive if $|x|=1$
$x=\pm 1$

Check end points
$x=1$ $\sum_{n=0}^{\infty} n(1)^n$ diverges
$x=-1$ $\sum_{n=0}^{\infty} n(-1)^n$ diverges

Interval of Convergence
$(-1,1)$
center $x=0$
radius $r=1$

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3. $\sum_{n=1}^{\infty}\frac{(x-2)^n}{n^3}$

$$\lim_{n\to\infty}\left|\frac{(x-2)^{n+1}}{(n+1)^3}\cdot\frac{n^3}{(x-2)^n}\right|$$

$$\lim_{n\to\infty}\left(|x-2|\cdot\frac{n^3}{(n+1)^3}\right)=|x-2|$$

Convergent if
$|x-2|<1$
$-1<x-2<1$
$1<x<3$

Check Endpoints
$x=1$ $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^3}$ conv.
$x=3$ $\sum_{n=1}^{\infty}\frac{(1)^n}{n^3}$ conv

Int. of Conv.
$[1,3]$
center 2
radius 1

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7. $\sum_{n=1}^{\infty}\frac{(2x)^n}{n^2}$

$$\lim_{n\to\infty}\left|\frac{(2x)^{n+1}}{(n+1)^2}\cdot\frac{n^2}{(2x)^n}\right|$$

$$\lim_{n\to\infty}\left|2x\cdot\frac{n^2}{(n+1)^2}\right|=|2x|$$

Converges if
$|2x|<1$
$-1<2x<1$
$-1/2<x<1/2$

Endpoints:
$x=1/2$ $\sum_{n=1}^{\infty}\frac{(1)^n}{n^2}$ converges
$x=-1/2$ $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$ converges

Int of Conv:
$[-1/2, 1/2]$
$c=0$ $r=1/2$

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11. $\sum_{n=0}^{\infty}(x/2)^n=\sum_{n=0}^{\infty}\frac{x^n}{2^n}$

$$\lim_{n\to\infty}\left|\frac{x^{n+1}}{2^{n+1}}\cdot\frac{2^n}{x^n}\right|$$

$$\lim_{n\to\infty}\left|\frac{\cancel{x^n}\cdot x^1\cdot\cancel{2^n}}{\cancel{2^n}\cdot 2\cdot\cancel{x^n}}\right|$$

$$\lim_{n\to\infty}|x/2|=\left|\frac{x}{2}\right|$$

Converges if
$|x/2|<1$
$-1<x/2<1$
$-2<x<2$

End points
$x=2$ $\sum_{n=0}^{\infty}(2/2)^n$ diverges
$x=-2$ $\sum_{n=0}^{\infty}(-2/2)^n$ diverges

Int of Conv:
$(-2,2)$
$c=0$ $r=2$

13. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1} x^{n+1}}{n+1} \cdot \frac{n}{(-1)^n x^n} \right|$$

$$\lim_{n\to\infty} \left| \frac{x^n \cdot x^1}{x^n} \cdot \frac{n}{n+1} \right| = |x|$$

Convergent if
$|x| < 1$
$-1 < x < 1$

Int of conv.
$(-1, 1]$
$c = 0 \quad r = 1$

End points:

$x = 1 \quad \sum_{n=1}^{\infty} \frac{(-1)^n (1)^n}{n}$ conv.

$x = -1 \quad \sum_{n=1}^{\infty} \frac{(-1)^n (-1)^n}{n}$

$\sum_{n=1}^{\infty} (1)^n/n$

$= \sum_{n=1}^{\infty} 1/n$ diverges

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15. $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

$$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} \right|$$

$$\lim_{n\to\infty} \left| \frac{\cancel{x^n} \cdot x \cdot \cancel{n!}}{(n+1)\cancel{n!}\cancel{x^n}} \right|$$

$$\lim_{n\to\infty} |x| \cdot \frac{1}{n+1} = 0$$

Convergent for all x since $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 0 < 1$

Interval of convergence

$(-\infty, \infty) \quad c = 0 \quad r = \infty$

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17. $\sum_{n=0}^{\infty} (2n)! \, (x/2)^n$

$$\sum_{n=0}^{\infty} \frac{(2n)! \, x^n}{2^n}$$

$$\lim_{n\to\infty} \left| \frac{(2(n+1))! \quad x^{n+1}}{2^{n+1}} \cdot \frac{2^n}{(2n)! \, x^n} \right|$$

$$\lim_{n\to\infty} \left| \frac{(2n+2)! \, \cancel{x^n} \cdot x \cdot \cancel{2^n}}{\cancel{2^n} \cdot 2 \cdot (2n)! \, \cancel{x^n}} \right|$$

$$\lim_{n\to\infty} \left| \frac{(2n+2)(2n+1)\cancel{(2n)!} \, x}{2 \, \cancel{(2n)!}} \right| = \infty$$

Divergent for all x except $x = 0$ since ratio test $= \infty > 1$

Convergent at $x = 0$

$\sum_{n=0}^{\infty} (2n)! \, (0/2)^n$

$\sum_{n=0}^{\infty} 0 = 0 + 0 + 0 = 0$

Interval of convergence
$x = 0 \quad$ center $= 0 \quad r = 0$

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25. $\sum_{n=1}^{\infty} \frac{(x-3)^{n-1}}{3^{n-1}}$

$$\lim_{n\to\infty} \left| \frac{(x-3)^n}{3^n} \cdot \frac{3^{n-1}}{(x-3)^{n-1}} \right|$$

$$\lim_{n\to\infty} \left| \frac{\cancel{(x-3)^n} \cdot \cancel{3^n} \cdot 3^{-1}}{\cancel{3^n} \cdot \cancel{(x-3)^n} \cdot (x-3)^{-1}} \right|$$

$$\lim_{n\to\infty} \left| \frac{x-3}{3} \right| = \left| \frac{x-3}{3} \right|$$

Convergent:

$\left| \frac{x-3}{3} \right| < 1$

$-1 < \frac{x-3}{3} < 1$

$-3 < x-3 < 3$

$0 < x < 6$

Int of conv:
$0 < x < 6 \quad c = 3 \quad r = 3$

Endpoints:

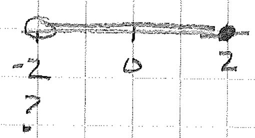
$x = 0 \quad \sum_{n=1}^{\infty} (-3/3)^{n-1}$

diverges

$x = 6 \quad \sum_{n=1}^{\infty} (3/3)^{n-1}$

diverges

79. $\sum_{n=0}^{\infty} a_n x^n$ center = 0



x = -2 would be an endpoint of the interval of convergence. It must be tested separately.

False it could converge at x = -2 but may not.

80. $\sum_{n=0}^{\infty} a_n x^n$ center = 0

$\boxed{\text{True}}$ x = -1 is in the interval.

p. 688: 23. $\sum_{n=0}^{\infty} (.82)^n$ converges geometric $|r| = .82 < 1$

24. $\sum_{n=0}^{\infty} (1.82)^n$ diverges geometric $|r| = 1.82 > 1$

25. $\sum_{n=1}^{\infty} \frac{(-1)^n (n)}{\ln(n)}$ $\lim_{n\to\infty} \frac{n}{\ln(n)} = \lim_{n\to\infty} \frac{1}{1/n} = \lim_{n\to\infty} n = \infty$

diverges nth term test

26. $\sum_{n=0}^{\infty} \frac{2n+1}{3n+1}$ diverges $\lim_{n\to\infty} \frac{2n+1}{3n+1} = \frac{2}{3} \neq 0$

nth term test

27. $\sum_{n=0}^{\infty} (2/3)^n$ geometric $a=1$ $r = 2/3$

$1 + 2/3 + 4/9 + \ldots$ sum $= \frac{1}{1-2/3} = \frac{1}{1/3} = \boxed{3}$