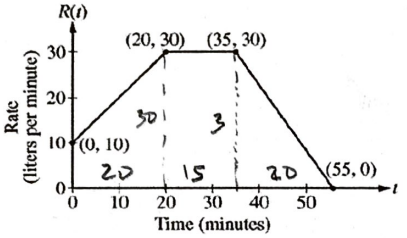
14. At time $t = 0$ minutes, a tank contains 100 liters of water. The piecewise-linear graph above shows the rate $R(t)$, in liters per minute, at which water is pumped into the tank during a 55-minute period.



(a) Find $R'(45)$. Using appropriate units, explain the meaning of your answer in the context of this problem.

(b) How many liters of water have been pumped into the tank from time $t = 0$ to time $t = 55$ minutes? Show the work that leads to your answer.

(c) At time $t = 10$ minutes, water begins draining from the tank at a rate modeled by the function D, where $D(t) = 10e^{(\sin t)/10}$ liters per minute. Water continues to drain at this rate until time $t = 55$ minutes. How many liters of water are in the tank at time $t = 55$ minutes?

(d) Using the functions R and D, determine whether the amount of water in the tank is increasing or decreasing at time $t = 45$ minutes. Justify your answer.

(a) $R'(45) = \frac{0-30}{55-35} = \frac{-30}{20} = \frac{-3}{2}$ liters/min^2

The rate at which water is being pumped into the tank at $t=45$ min is decreasing by $-3/2$ liters/min^2.

(b) $\int_0^{55} R(t)\,dt = \frac{20(10+30)}{2} + 15(30) + \frac{1}{2}(20)(30)$

$= 400 + 450 + 300 = 1150$

(c) $A(t)$ is amount in tank at time t.

$A(0) = 100$

$A(t) = A(0) + 1150 - \int_{10}^{55} D(t)\,dt$

$= 100 + 1150 - 450.275371 = 799.725$

or 799.724

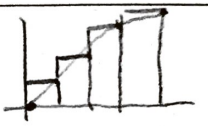
(d) $R(45) - D(45)$

$15 - 10.888315 > 0$

water in tank is inc at $t=45$ since $R(45) - D(45) > 0$.

15.

t (seconds)	0	3	5	8	12
$k(t)$ (feet per second)	0	5	10	20	24



Kathleen skates on a straight track. She starts from rest at the starting line at time $t = 0$. For $0 < t \le 12$ seconds, Kathleen's velocity k, measured in feet per second, is differentiable and increasing. Values of $k(t)$ at various times t are given in the table above.

(a) Use the data in the table to estimate Kathleen's acceleration at time $t = 4$ seconds. Show the computations that lead to your answer. Indicate units of measure.

(b) Use a right Riemann sum with the four subintervals indicated by the data in the table to approximate $\int_0^{12} k(t)\,dt$. Indicate units of measure. Is this approximation an overestimate or an underestimate for the value of $\int_0^{12} k(t)\,dt$? Explain your reasoning.

(c) Nathan skates on the same track, starting 5 feet ahead of Kathleen at time $t = 0$. Nathan's velocity, in feet per second, is given by $n(t) = \frac{150}{t+3} - 50e^{-t}$. Write, but do not evaluate, an expression involving an integral that gives Nathan's distance from the starting line at time $t = 12$ seconds.

(d) Write an expression for Nathan's acceleration in terms of t.

(a) $a(4) \approx \frac{k(5)-k(3)}{5-3} = \frac{10-5}{2} = \frac{5}{2}$ ft/sec^2

(b) $\int_0^{12} k(t)\,dt = 3(5) + 2(10) + 3(20) + 4(24) = 191$ ft

191 ft is an over-estimate since k is inc.

(c) $n(t) = \frac{150}{t+3} - 50e^{-t} = 150(t+3)^{-1} - 50e^{-t}$

Nathan's distance from starting line

$5 + \int_0^{12} n(t)\,dt$

(d) $n'(t) = -150(t+3)^{-2} \cdot 1 + 50e^{-t}$

$= \frac{-150}{(t+3)^2} + \frac{50}{e^t}$