9-#11 Part II

1. $\lim_{x\to 0} \frac{\sin(x^3)-x^3}{x^9}$

$\lim_{x\to 0} \frac{x^3 - \frac{x^9}{3!} + \frac{x^{15}}{5!} - \frac{x^{21}}{7!} + \cdots}{x^9}$

$\lim_{x\to 0} \frac{1}{x^6} - \frac{1}{3!} + \frac{x^6}{5!} + \cdots = -\frac{1}{3!} = \boxed{\frac{-1}{6}}$

To use L'Hopital 9 derivs would be needed

2. $\int_{-1}^{1} \cos(x^2)\,dx$

$2\int_0^1 1 - \frac{x^4}{2!} + \frac{x^8}{4!}\,dx$

$2\left[x - \frac{1}{5}\frac{x^5}{2} + \frac{1}{9}\frac{x^9}{24}\right]_0^1$

$2\left[1 - \frac{1}{10} + \frac{1}{216}\right]$

$\boxed{2 - 1/5 + 1/108} = \frac{977}{540}$

3. $f(x) = \ln(3-x)$ $\quad f(2) = 0$

$f'(x) = -(3-x)^{-1}$ $\quad f'(2) = -1$

$f''(x) = -(3-x)^{-2}$ $\quad f''(2) = -1$

$f'''(x) = -2(3-x)^{-3}$ $\quad f'''(2) = -2$

$\frac{f(2)(x-2)^0}{0!} + \frac{f'(2)(x-2)^1}{1!} + \frac{f''(2)(x-2)^2}{2!} \cdots$

$0 - 1(x-2) - \frac{(x-2)^2}{2!} + \frac{2(x-2)^3}{3!}$

$-(x-2) - \frac{(x-2)^2}{2} - \frac{(x-2)^3}{3}$ B

4. $1 \quad x - \frac{x^3}{3!} + \frac{x^5}{5!} = \sin(x)$

$x - 1/3! + \frac{1}{5!} + \ldots = \sin(1)$

$1 - 1/6 + 1/120$ E

5. $f'(x) = \sin(x^2)$

$= \frac{(x^2)}{1!} - \frac{x^6}{3!} + \frac{x^{10}}{5!}$

$f(x) = 1/3\,x^3 - \frac{1}{7}\frac{x^7}{3!} + \frac{1}{11}\frac{x^{11}}{5!}$

$= 1/3\,x^3 - \frac{1}{42}x^7 + \frac{1}{11\cdot 5!}x^{11}$

D

6. $\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}}$

$\lim_{n\to\infty}\left[\frac{(x+2)^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{(x+2)^n}\right]$

$\lim_{n\to\infty}\left[(x+2)\frac{\sqrt{n}}{\sqrt{n+1}}\right| = |x+2|$

Conv if $|x+2| < 1$

$-1 < x+2 < 1$

$-3 < x < -1$

$x = -3 \quad \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ conv. AST

$x = -1 \quad \sum_{n=1}^{\infty} \frac{(1)^n}{\sqrt{n}}$ diverges p-series

Int of Conv $-3 \le x < -1$ B

7. $\frac{x^2}{1-x^2}$ $\quad a = x^2$ $\quad r = x^2$

$x^2 + x^4 + x^6 + x^8$ D

8. $\frac{x^4}{2!} + \frac{x^5}{3!} + \frac{x^6}{4!} + \ldots + \frac{x^{n+3}}{(n+1)!}$

$x^2\left(\frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^{n+1}}{(n+1)!}\right)$

↑ almost $e^x$ but missing first two terms add them in and subtract them out.

$x^2\left[\left(1 + \frac{x}{1} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\right) - 1 - x\right]$

$x^2\left[e^x - 1 - x\right]$

$x^2 e^x - x^2 - x^3$ D

9. $f(x) = (1+x)^2$ $\quad f(0) = 1$

$f'(x) = -2(1+x)^{-3}$ $\quad f'(0) = -2$

$f''(x) = 6(1+x)^{-4}$ $\quad f''(0) = 6$

$$\frac{f(0)x^0}{0!} + \frac{f'(0)x^1}{1!} + \dots$$

$$1 - \frac{2x}{1} + \frac{6x^2}{2!}$$

$\frac{6}{2!} = 3$ **D**

10. $f(x) = 3x^2 - 5x^3 + 7x^4 + 3x^5$

$$\frac{f'''(0)x^3}{3!}$$

$\frac{f'''(0)}{3!} = -5$

$f'''(0) = -5 \cdot 3!$

$= -30$ **A**

11. $f(x) = \cos(x)$ $\quad f(\pi/4) = \sqrt{2}/2$

$f'(x) = -\sin(x)$ $\quad f'(\pi/4) = -\sqrt{2}/2$

$f''(x) = -\cos(x)$ $\quad f''(\pi/4) = -\sqrt{2}/2$

$f'''(x) = \sin x$ $\quad f'''(\pi/4) = \sqrt{2}/2$

$$\frac{f(\pi/4)(x-\pi/4)^0}{0!} + \frac{f'(\pi/4)(x-\pi/4)^1}{1!} + \dots$$

$$\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}(x-\pi/4)^1 - \frac{\sqrt{2}}{2}\frac{(x-\pi/4)^2}{2!} + \frac{\sqrt{2}}{2}\frac{(x-\pi/4)^3}{3!}$$

$\frac{\sqrt{2}}{2 \cdot 3!} = \frac{\sqrt{2}}{12}$ **D**

12. $f(x) = x\sin(2x)$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}$

$\sin(2x) = (2x) - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \frac{(2x)^7}{7!}$

$x\sin(2x) = 2x^2 - \frac{8x^4}{3!} + \frac{32x^6}{5!} - \frac{128x^8}{7!}$ **E**

13. $f(3) = 2$ $\quad f'(3) = -1$

$f''(3) = 6$ $\quad f'''(3) = 12$

$$\frac{f(3)(x-3)^0}{0!} + \frac{f'(3)(x-3)^1}{1!} + \dots$$

$$2 - (x-3) + \frac{6(x-3)^2}{2!} + \frac{12(x-3)^3}{3!} \dots$$

$2 - (x-3) + 3(x-3)^2 + 2(x-3)^3$ **A**

Answers:

1. $-1/6$
2. $2 - 1/5 + 1/108$
3. B
4. E
5. D
6. B
7. D
8. D
9. D
10. A
11. D
12. E
13. A