9 - 3: p. 620: 1, 3, 5, 21, 25, 27, 29, 33, 83, 84, 85, 87, 88, 89

1. $\sum_{n=1}^{\infty} \frac{1}{n+1}$ decreasing ✓ positive ✓ continuous ✓

$\lim_{b\to\infty} \int_1^b \frac{1}{x+1}\,dx$

$\lim_{b\to\infty} \left[\ln|x+1|\right]_1^b$

$\lim_{b\to\infty} \ln|b+1| - \ln(2)$

$= \infty$

Integral diverges so $\sum_{n=1}^{\infty} 1/n+1$ diverges

3. $\sum_{n=1}^{\infty} e^{-n}$ decreasing ✓ pos ✓ cont ✓

$\lim_{b\to\infty} \int_1^b e^{-x}\,dx$

$\lim_{b\to\infty} \left[-e^{-x}\right]_1^b$

$\lim_{b\to\infty} \left[-e^{-b} + e^{-1}\right]$

$\lim_{b\to\infty} \left[\frac{-1}{e^b} + \frac{1}{e}\right] = 1/e$

Integral converges so series converges

note: $\sum_{n=1}^{\infty} e^{-n} \neq \frac{1}{e}$

5. $1/2 + 1/5 + 1/10 \dots \frac{1}{n^2+1}$ ← decreasing ✓ pos ✓ cont ✓

$\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges since integral converges

$\lim_{b\to\infty} \int_1^b \frac{1}{x^2+1}\,dx$

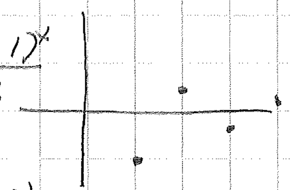
$\lim_{b\to\infty} \left[\tan^{-1}x\right]_1^b$

$\lim_{b\to\infty} \tan^{-1}(b) - \tan^{-1}(1) = \pi/2 - \pi/4 = \pi/4$

21. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} = -1 + 1/2 - 1/3$

If $f(x) = \frac{(-1)^x}{x}$

then $f(x)$ is not positive nor decreasing for all x.



25. $\sum_{n=1}^{\infty} 1/n^3$ ← dec ✓, pos ✓, cont ✓

$\lim_{b\to\infty} \int_1^b x^{-3}\,dx$

$\lim_{b\to\infty} \left[-\tfrac{1}{2}\, 1/x^2\right]_1^b$

$\lim_{b\to\infty} \left[-1/2x^2 + 1/2\right] = 1/2$

Integral converges so series converges

27. $\sum_{n=1}^{\infty} 1/\sqrt{n}$ ← dec ✓, pos ✓, cont ✓

$\lim_{b\to\infty} \int_1^b x^{1/2}\,dx$

$\lim_{b\to\infty} \left[\tfrac{2}{3} x^{3/2}\right]_1^b$

$\lim_{b\to\infty} \tfrac{2}{3} b^{3/2} - \tfrac{2}{3} = \infty$

Integral diverges so series diverges

29. $\sum_{n=1}^{\infty} 1/\sqrt[5]{n} = \sum_{n=1}^{\infty} 1/n^{1/5}$

Co[illegible]erges p-series $p = 1/5 < 1$

33. $1 + \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} + \cdots$ $\quad 3^1 \cdot 3^{1/2} = 3^{3/2}$

$\frac{1}{1^{3/2}} + \frac{1}{2^{3/2}} + \cdots$

$$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$$

diverges p-series
$p = 3/2 > 1$

83. $\sum_{n=0}^{\infty} (2/3)^n$

converges
geometric
$|r| = 2/3 < 1$

84. $\sum_{n=0}^{\infty} (1.075)^n$

diverges
geometric
$|r| = 1.075 > 1$

85. $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$

$\lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = 1 \neq 0$

diverges nth term
test

87. $\sum_{n=1}^{\infty} (1 + 1/n)^n$

$\lim_{n\to\infty} (1+1/n)^n = e \neq 0$

diverges, nth
term test

88. $\sum_{n=2}^{\infty} \ln(n)$

$\lim_{n\to\infty} \ln(n) = \infty \neq 0$

diverges, nth term
test

89. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$

$f(x) = \frac{1}{x(\ln(x))^3} dx$ $\quad$ pos ✓ dec ✓ cont ✓

$\int_{\ln 2}^{\infty} \frac{1}{u^3} du$ $\quad$ $u = \ln(x)$, $du = 1/x \, dx$

$\left[-\tfrac{1}{2} u^{-2} \right]_{\ln 2}^{\infty}$

$0 - (-\tfrac{1}{2} \ln^{-2}(2))$

$= \frac{1}{2\ln^2(2)}$

Converges by integral
test.

(if integral converges
series converges)