9-#9

1. $f(x) = x^{-1}$ $\quad f(1) = 1$

$f'(x) = -x^{-2}$ $\quad f'(1) = -1$

$f''(x) = 2x^{-3}$ $\quad f''(1) = 2$

$f'''(x) = -6x^{-4}$ $\quad f'''(1) = -6$

$f^{(4)}(x) = 24x^{-5}$ $\quad f^{(4)}(1) = 24$

$$P_4(x) = \frac{f(1)(x-1)^0}{0!} + \frac{f'(1)(x-1)^1}{1!} + \frac{f''(1)(x-1)^2}{2!} \dots$$

$$P_4(x) = 1 - (x-1)^1 + \frac{2(x-1)^2}{2} - \frac{6(x-1)^3}{3!} + \frac{24(x-1)^4}{4!}$$

$$P_4(x) = 1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4$$

2. $f(x) = \sin(x)$ $\quad f(\pi) = 0$

$f'(x) = \cos(x)$ $\quad f'(\pi) = -1$

$f''(x) = -\sin(x)$ $\quad f''(\pi) = 0$

$f'''(x) = -\cos(x)$ $\quad f'''(\pi) = 1$

$f^{(4)}(x) = \sin(x)$ $\quad f^{(4)}(\pi) = 0$

$f^{(5)}(x) = \cos(x)$ $\quad f^{(5)}(\pi) = -1$

$$P_5(x) = \frac{f(\pi)(x-\pi)^0}{0!} + \frac{f'(\pi)(x-\pi)^1}{1!} + \frac{f''(x)(x-\pi)^2}{2!} + \dots$$

$$P_5(x) = 0 - (x-\pi) + 0 + \tfrac{1}{6}(x-\pi)^3 + 0 - \tfrac{1}{120}(x-\pi)^5$$

$$P_{(5)} = -(x-\pi) + \tfrac{1}{6}(x-\pi)^3 - \tfrac{1}{120}(x-\pi)^5$$

3. a) $P_3(3.7) = -(2.4-\pi) + \tfrac{1}{6}(2.4-\pi)^3$

$= .6736$

b) $|\text{Error}| \le |\text{first neglected term}|$

$|\text{error}| \le \left|-\tfrac{1}{120}(2.4-\pi)^5\right|$

$|\text{error}| \le .0018691573$

The $\sin(2.4) \approx P(2.4)$ the error bounded by $.0018691573$ so

$P_3(2.4) - |\text{error}| \le \sin(2.4) \le P_3(2.4) + |\text{error}|$

c) $0.671749 \le \sin(2.4) \le 0.67548$

d) $\max|f^{(5)}(z)|$ for $2.4 \le z \le \pi$

$\max|\cos(z)| = 1$

$\boxed{\text{on calc}}$ → Check: $\sin(2.4) = .6754631806$

4. $f(x) = \sqrt{x}$ $\qquad f(1) = 1$

$f'(x) = 1/2\, x^{-1/2} \qquad f'(1) = 1/2$

$f''(x) = -1/4\, x^{-3/2} \qquad f''(1) = -1/4$

$f'''(x) = 3/8\, x^{-5/2} \qquad f'''(1) = 3/8$

$f^{(4)}(x) = -15/16\, x^{-7/2} \qquad f^{(4)}(1) = -15/16$

$\frac{5\cdot\cancel{3}}{16} \quad \frac{1}{4\cdot\cancel{3}\cdot 2\cdot 1}$

$$P_4(x) = 1 + 1/2(x-1)^1 - 1/4 \cdot 1/2!\,(x-1)^2 + 3/8 \cdot 1/3!\,(x-1)^3 - 15/16\, 1/4!\,(x-1)^4$$
$$= 1 + 1/2(x-1) - 1/8(x-1)^2 + 1/16(x-1)^3 - 1/128(x-1)^4$$

5. a) $P_4(0.7) = 0.8370625$

b) $|\text{Error}| \le \frac{|\max f^{(4)}(z)|\,|.7-1|^4}{4!}$

$|\text{Error}| \le \frac{3.267 \cdot (.3)^4}{24}$

$|\text{Error}| \le .001102$

c) $P_4(.7) - |\text{error}| \le \sqrt{.7} \le P_4(.7) + |\text{error}|$

$.8370625 - .001102 \le \sqrt{.7} \le .8370625 + .001102$

$.8359599408 \le \sqrt{.7} \le .8381650592$

$.83596 \le \sqrt{.7} \le .83817$

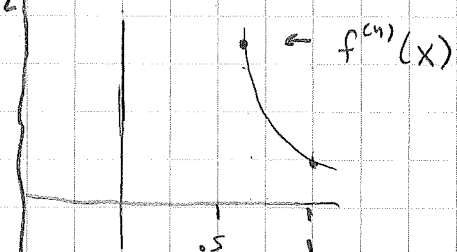
Check: $\sqrt{.7} = .8366600265$ ☺

where z is the x value of the max of $f^{(4)}$ on $[.7, 1]$

$f^{(4)}(x) = \frac{-15}{16x^{7/2}}$

$|f^{(4)}(.7)| = 3.267$

$|f^{(4)}(1)| = .9375$



6. $f(x) = 2x^{-2} \qquad f(2) = 1/2$

$f'(x) = -4x^{-3} \qquad f'(2) = -1/2$

$f''(x) = 12x^{-4} \qquad f''(2) = 3/4$

$f'''(x) = -48x^{-5} \qquad f'''(2) = -3/2$

$$P_3(x) = \frac{f(2)(x-2)^0}{0!} + \frac{f'(2)(x-2)^1}{1!} + \frac{f''(2)(x-2)^2}{2!} + \frac{f'''(2)(x-2)^3}{3!}$$

$$= \frac{1}{2} - \frac{1}{2}(x-2) + \frac{3}{4}\cdot\frac{1}{2}(x-2)^2 - \frac{3}{2}\cdot\frac{1}{6}(x-2)^3$$

$$= \frac{1}{2} - \frac{1}{2}(x-2) + \frac{3}{8}(x-2)^2 - \frac{1}{4}(x-2)^3$$