Ch. 3 Review Solutions

1. $g(x) = 4f(x)\sin(x)$

$g'(x) = 4\cdot[f(x)\cdot\cos(x) + f'(x)\sin(x)]$

$g'(\pi/6) = 4[f(\pi/6)\cos(\pi/6) + f'(\pi/6)\sin(\pi/6)]$

$= 4[3\cdot\sqrt{3}/2 + (-4)(1/2)]$

$= 4[3\sqrt{3}/2 - 2]$

$= 6\sqrt{3} - 8$ $\boxed{B}$

2. $f(x) = x^3 - 7x + 6$

$f'(x) = 3x^2 - 7$

$\frac{f(3) - f(1)}{3-1} = \frac{12-0}{2} = 6$

$3x^2 - 7 = 6$

$3x^2 = 13$

$x^2 = 13/3$

$x = \sqrt{13/3} = \sqrt{13}/\sqrt{3}$

$= \sqrt{39}/3$ $\boxed{C}$

3. $y = (x+1)\arctan(x)$

$y' = (x+1)\left(\frac{1}{1+x^2}\right) + (1)\arctan(x)$

$y' = \frac{x+1}{1+x^2} + \arctan(x)$

$y''(x) = \frac{(1+x^2)(1) - (x+1)(2x)}{(1+x^2)^2} + \frac{1}{1+x^2}$

$= \frac{1+x^2-2x^2-2x}{(1+x^2)^2} + \frac{1}{(1+x^2)}\cdot\frac{(1+x^2)}{(1+x^2)}$

$= \frac{x^2-2x^2+x^2-2x+1+1}{(1+x^2)^2}$

$= \frac{-2x+2}{(1+x^2)^2}$

$\xleftarrow{\quad + \quad | \quad - \quad} y''$ $\boxed{E}$

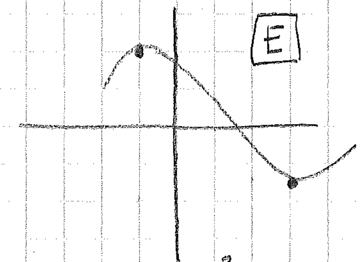
$y''(x) = 0$

$-2x + 2 = 0$

$-2x = -2$ $\boxed{x = 1}$

$y = (1+1)\arctan(1)$

$y = 2(\pi/4) = \boxed{\pi/2}$

4.  $\boxed{E}$

5. $f(x) = (1+\tan x)^{3/2}$

$f(0) = (1+0)^{3/2} = 1$

$f'(x) = 3/2(1+\tan x)^{1/2}\cdot\sec^2 x$

$f'(0) = 3/2(1+0)^{1/2}\cdot(1)^2$

$= 3/2(1)(1) = 3/2$

$y - 1 = 3/2(x - 0)$

$y = 3/2x + 1$

$L(x) = 3/2x + 1$ ← Local Linear approximation of f(x) near x = 0

$L(.02) = \frac{3}{2}(.02) + 1$

$= 3(.01) + 1$

$0.03 + 1 = \boxed{1.03}$

$\boxed{A}$

6. $h(x) = f^2(x) - g^2(x)$

$h'(x) = 2f(x)f'(x) - 2g(x)\cdot g'(x)$

$= 2f(x)\cdot -g(x) - 2g(x)\cdot f(x)$

$= -2f(x)g(x) - 2f(x)g(x)$

$= -4f(x)g(x)$

$\boxed{C}$

7. $f(x) = x^3 - 3x^2 + 12$

$f'(x) = 3x^2 - 6x$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0 \quad x = 2$ $\boxed{A}$

$f(-2) = -8 - 12 + 12 = -8$ min

$f(0) = 12$

$f(2) = 8 - 12 + 12 = 8$

$f(4) = 64 - 48 + 12 = 28$ max

8. constraint: $\pi r^2 h = 16\pi$ in^3

optimize: $A = 2\pi r^2 + 2\pi r h$

$A = 2\pi r^2 + 2\pi r(16/r^2)$ $\quad h = 16/r^2$

$= 2\pi r^2 + 32\pi/r$

$A'(r) = 4\pi r - 32\pi r^{-2}$

$= \frac{4\pi r^3 - 32\pi}{r^2}$

$4\pi r^3 - 32\pi = 0$

$4\pi r^3 = 32\pi$

$r^3 = 8$

$r = 2$

$h = 16/2^2 = 4$

$\boxed{D}$

9. $f(x) = x\ln(x)$

$f'(x) = x \cdot \frac{1}{x} + \ln(x) \cdot 1$

$= 1 + \ln(x)$

$1 + \ln(x) = 0$

$\ln(x) = -1$

$x = e^{-1}$

$x = 1/e$

$-$ $+$ f'

$1/e$

$f(1/e) = 1/e \cdot \ln(1/e)$

$= \frac{1}{e} \cdot -1$

$= -1/e$ $\boxed{C}$

The minimum is at

$(1/e, -1/e)$

← minimum

10. $f(x) = (x-2)(x-3)^2$

$f'(x) = (x-2) \cdot 2(x-3) + (1)(x-3)^2$

$= (x-3)[2(x-2) + 1(x-3)]$

$= (x-3)[2x-4+x-3]$

$= (x-3)(3x-7) = 0$

$x = 3 \quad x = 7/3$

$+$ $-$ $+$ f'

$7/3$ 3

$\boxed{D}$

11. $f(x) = (x^2-3)e^{-x}$

$f'(x) = (2x)e^{-x} + (x^2-3) \cdot e^{-x} \cdot -1$

$= e^{-x}(2x - (x^2-3))$

$= e^{-x}(2x - x^2 + 3)$

$= -e^{-x}(x^2 - 2x - 3)$

$= \frac{-1(x-3)(x+1)}{e^x}$

dec inc dec

$-$ $+$ $-$ $f'(x)$

-1 3

$\boxed{D}$

e^x never $= 0$ and is always positive.

12. $\lim_{x\to 0} \frac{e^{2x}-1}{\tan x}$ $\boxed{D}$

$\lim_{x\to 0} \frac{2e^{2x}}{\sec^2 x} = \frac{2e^0}{1^2} = 2$

13. $\lim_{x\to 0} \frac{2\sin(x) - \sin(2x)}{x - \sin x}$ $\frac{0}{0}$

LH $\left(\lim_{x\to 0} \frac{2\cos(x) - 2\cos(2x)}{1-\cos(x)} \right.$ $\frac{0}{0}$

LH $\left(\lim_{x\to 0} \frac{-2\sin x + 4\sin(2x)}{\sin x} \right.$ $\frac{0}{0}$ $\boxed{B}$

LH $\left(\lim_{x\to 0} \frac{-2\cos(x) + 8\cos(2x)}{\cos(x)} \right.$ $= \frac{-2+8}{1} = 6$

14. $f(x) = e^x$

$f'(x) = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h}$

$f'(e) = \lim_{h\to 0} \frac{e^{e+h} - e^e}{h}$ $\boxed{E}$

15. $\lim_{x\to 0} x\csc x$ $\boxed{D}$

$\lim_{x\to 0} \frac{x}{\sin x} \overset{LH}{=} \lim_{x\to 0} \frac{1}{\cos(x)} = 1$

16. $\frac{dr}{dt} = .3$ in/sec Find $\frac{dV}{dt}$ is when $S = 100\pi$

$V = 4/3\pi r^3$ $4\pi r^2 = 100\pi$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ $r^2 = 25$

$r = 5$

$= 4\pi(5)^2(.3)$

$= 100\pi \cdot \frac{3}{10}$

$= 30\pi$ $\boxed{E}$

17. $x(t) = -5t^2$

$\frac{x(3) - x(0)}{3-0} = \frac{-45-0}{3} = -15$ $\boxed{C}$

18. $x(t) = te^{-2t}$

$v(t) = x'(t)$

$= t \cdot e^{-2t} \cdot -2 + 1 \cdot e^{-2t}$

$= e^{-2t}(-2t+1)$

$v(t) = 0$ object at rest

$e^{-2t}(-2t+1) = 0$

$e^{-2t} \neq 0 \quad -2t+1 = 0$

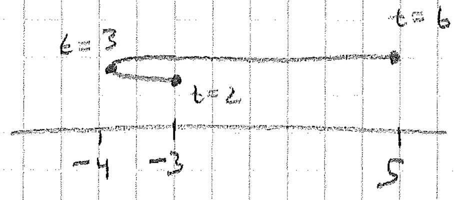
$-2t = -1$

$t = 1/2$ $\boxed{C}$

19. $x(t) = t^2 - 6t + 5 \quad [2, 6]$

$x'(t) = v(t)$
$= 2t - 6 = 0$
$2t = 6$
$t = 3$

$x(2) = -3$ > 1
$x(3) = -4$ > 9
$x(6) = 5$



Total Distance $= 1 + 9$
$= 10$

20. If I is $f(x)$ then $f'(0)$ must be negative. so I is not $f(x)$.

If II is $f(x)$ then $f'(0) = 0$ so III could be $f'(x)$.

If III is $f'(x)$ then $f''(0) =$ positive and I has a positive value at $x = 0$.

$\boxed{C}$

21. $f(x) = 2x^4 - 5x^6$
$f'(x) = 8x^3 - 30x^5$
$f''(x) = 24x^2 - 150x^4$
$= 6x^2(4 - 25x^2) = 0$
$6x^2 = 0 \quad 4 - 25x^2 = 0$
$x = 0 \quad -25x^2 = -4$
$x^2 = 4/25$
$x = \pm 2/5$

	$-2/5$		0		$2/5$		
$-$		$+$		$+$		$-$	f''

$\boxed{B}$

22. $f(x) = \sin(x/2)$
$f'(x) = \cos(x/2) \cdot 1/2$

$\dfrac{f(3\pi/2) - f(\pi/2)}{3\pi/2 - \pi/2} = \dfrac{\sin(3\pi/4) - \sin(\pi/4)}{3\pi/2 - \pi/2}$

$= \dfrac{\sqrt{2}/2 - \sqrt{2}/2}{2\pi/2} = \dfrac{0}{\pi} = 0$

22 cont'd

$1/2 \cos(x/2) = 0$
$\cos(x/2) = 0$
$\dfrac{x}{2} = \dfrac{\pi}{2} + \pi k$

$x = \pi + 2\pi k$

$x = \pi$ $\boxed{D}$

FRQ

23.

-7		-5		-1		0		5		
	$+$	0	$-$	0	$+$			0	$+$	f'
	inc		dec		inc				inc	

a) rel min at $x = -1$ since $f'(x)$ changes from neg to pos at $x = -1$

b) rel max at $x = -5$ since $f'(x)$ changes from pos to neg at $x = -5$.

c)

-7		-3		0		2		3		5	7	
	$-$	0	$+$			0	$-$	u	$-$	0	$+$	$f''(x)$
	CD		CU				CD		CD		CU	

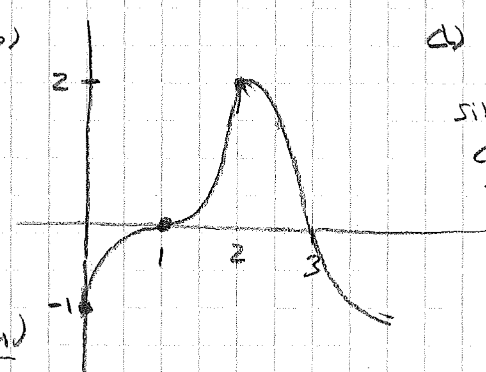
$f''(x) < 0$ for

$(-7, -3) \quad (2, 3) \quad (3, 5)$

24.

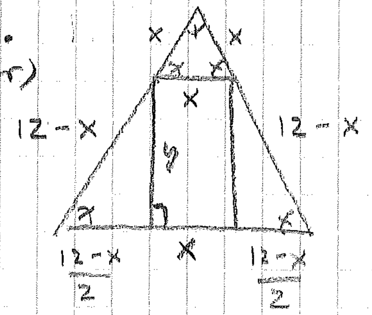
0		1		2		3		4		
	$-$	0	$+$			0			$-$	f
	$+$	0	$+$	u			$-$			f'
	inc		inc			dec				
	$-$	0	$+$	u	$-$	0	$+$			f''
	CD		CU		CD		CU			

b)



d) rel max at $x = 2$ since $f(x)$ changes from pos to neg at $x = 2$.

25. (easier)



Constraint:

$$\left(\frac{12-x}{2}\right)^2 + y^2 = (12-x)^2$$

$$\frac{(12-x)^2}{4} + y^2 = (12-x)^2$$

$$y^2 = \frac{(12-x)^2}{1} - \frac{(12-x)^2}{4}$$

$$y^2 = \frac{4(12-x)^2}{4} - \frac{(12-x)^2}{4}$$

$$y^2 = \frac{3(12-x)^2}{4}$$

$$y = \frac{\sqrt{3}\,(12-x)}{2}$$

Maximize:

$$A = xy$$

$$A = x\left(\frac{\sqrt{3}}{2}(12-x)\right)$$

$$A = \frac{\sqrt{3}}{2}(12x - x^2)$$

$$A' = \frac{\sqrt{3}}{2}(12-2x)$$

$$12 - 2x = 0$$

$$12 = 2x$$

$$\boxed{6 = x}$$

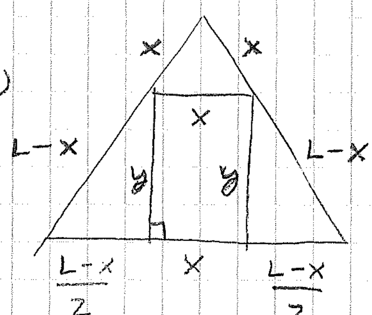
$$y = \frac{\sqrt{3}\,(12-6)}{2}$$

$$y = \frac{\sqrt{3}\,(6)}{2}$$

$$\boxed{y = 3\sqrt{3}}$$

max area $= 18\sqrt{3}$

25. (harder)



Constraint:

$$\left(\frac{L-x}{2}\right)^2 + y^2 = (L-x)^2$$

$$y^2 = \frac{(L-x)^2}{1} - \frac{(L-x)^2}{4}$$

$$y^2 = \frac{4(L-x)^2}{4} - \frac{(L-x)^2}{4}$$

$$y^2 = \frac{3(L-x)^2}{4}$$

$$y = \frac{\sqrt{3}\,(L-x)}{2}$$

Maximize

$$A = xy$$

$$A = x\left(\sqrt{3}/2\,(L-x)\right)$$

$$= \sqrt{3}/2\,(Lx - x^2)$$

$$A' = \sqrt{3}/2\,(L - 2x)$$

$$A' = 0$$

$$L - 2x = 0$$

$$L = 2x$$

$$\boxed{\frac{L}{2} = x}$$

$$y = \frac{\sqrt{3}\,(L - L/2)}{2}$$

$$= \frac{\sqrt{3}\,(L/2)}{2} = \boxed{\frac{\sqrt{3}}{4}L}$$

Max Area =

$$\frac{L}{2} \cdot \frac{\sqrt{3}}{4}L$$

$$\boxed{\frac{\sqrt{3}}{8}L^2}$$