6.1 Eulers Method HW

1. $\frac{dy}{dx} = y^2(2x+2)$ $f(0) = -1$ $(0,-1)$

a) $\lim_{x \to 0} \frac{f(x)+1}{\sin x}$

$\lim_{x \to 0} \frac{f'(x)}{\cos x}$

$= \frac{f'(0)}{\cos(0)}$

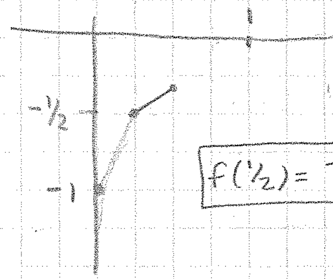
$= \frac{(-1)^2(2 \cdot 0 + 2)}{1} = \boxed{2}$

constant step size

$\frac{dy}{dx} \cdot \Delta x$

b)

x	y	m dy/dx	Δx	Δy	$y + \Delta y$
0	-1	2	$1/4$	$1/2$	$-1/2$
$1/4$	$-1/2$	$5/8$	$1/4$	$5/32$	$-11/32$
$1/2$	$-11/32$				



$\boxed{f(1/2) = -11/32}$

$\frac{dy}{dx}\Big|_{(1/4,\,-1/2)} = \left(-\frac{1}{2}\right)^2 (2 \cdot 1/4 + 2)$

$= \frac{1}{4}\left(\frac{1}{2} + \frac{4}{2}\right)$

$\frac{1}{4}\left(\frac{5}{2}\right) = 5/8$

c) $y^{-2}\, dy = (2x+2)\, dx$

$-y^{-1} = x^2 + 2x + c$

$y^{-1} = -x^2 - 2x + c$

$y = \frac{1}{-x^2-2x+c}$

$-1 = \frac{1}{0+0+c}$

$-1c = 1$

$c = -1$

$y = \frac{1}{-x^2-2x-1}$

$\boxed{y = \frac{-1}{x^2+2x+1}}$

3c. $\ln|1-y| = -x+1$

$|1-y| = e^{1-x}$

$1-y = \pm e^{1-x}$

$-y = \pm e^{1-x} - 1$

$y = \pm e^{1-x} + 1$

$\boxed{y = 1 - e^{1-x}}$ $(1,0)$

2.

x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

$f(1) = 15$ $f''(1) = 20$

a) $f'(1) = 8$

$f(1) = 15$

$y = 8(x-1) + 15$

$f(1.4) \approx 8(1.4-1) + 15$

$8(.4) + 15$

$3.2 + 15 = \boxed{18.2}$

b) $\int_1^{1.4} f'(x)\,dx = (.2)(10) + (.2)(13)$

$= 2 + 2.6$

$= 4.6$

$f(1.4) = f(1) + \int_1^{1.4} f'(x)\,dx$

$= 15 + 4.6$

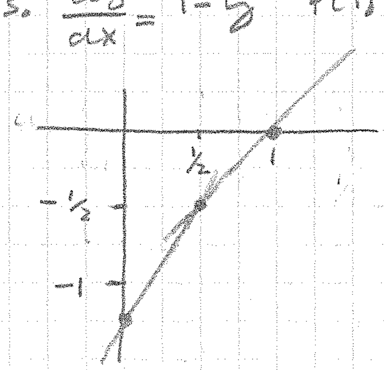
$= \boxed{19.6}$

c)

x	y	m	Δx	Δy	New point
1	15	8	.2	1.6	(1.2, 16.6)
1.2	16.6	12	.2	2.4	(1.4, 19.0)
1.3	19.0				

$\boxed{19.0}$

3. $\frac{dy}{dx} = 1 - y$ $f(1) = 0$ $f(x) < 1$



$f(x_1) = f(x_0) + m_0 \Delta x$

$f(1/2) = f(1) + (1)(-1/2)$

$= 0 - 1/2$

$= -1/2$

$f(0) = f(1/2) + 3/2(-1/2)$

$= -1/2 - 3/4$

$= -5/4$

b) $\lim_{x \to 1} \frac{f(x)}{x^3-1} = \lim_{x \to 1} \frac{f'(x)}{3x^2}$

$= \frac{f'(1)}{3(1)^2} = \frac{1}{3}$

c) $\frac{1}{1-y}\, dy = dx$

$\ln|1-y| \cdot -1 = x + c$

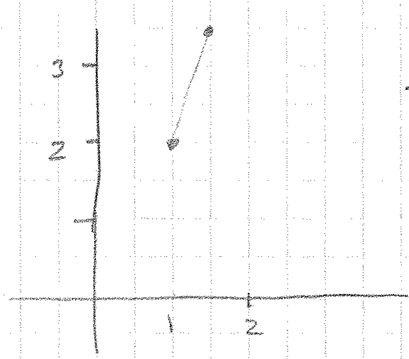
$\ln|1-y| = -x + c$ $(1,0)$

$\ln|1| = -1 + c$

$1 = c$

4. $y = f(x)$ $\frac{dy}{dx} = x + y$ $f(1) = 2$

$f(2) = ?$ $f(1)$ step $.5$



$$f(1.5) = f(1) + m \cdot \Delta x = 2 + 3 \cdot (.5) = 2 + 1.5 = 3.5$$

$$f(2) = f(1.5) + m\Delta x = 3.5 + 5(.5) = 3.5 + 2.5 = 6$$ $\boxed{C}$

x	y	m	Δx	Δy	y+Δx
1	2	3	.5	1.5	3.5
1.5	3.5	5	.5	2.5	6
2	<u>6</u>				

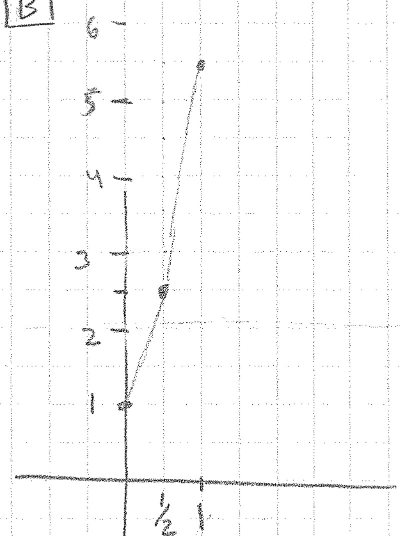
5. $\frac{dy}{dx} = x - y - 1$ $f(1) = -2$ $\Delta x = .2$

$f(1.2) = f(1) + m_1 \Delta x = -2 + 2(.2) = -2 + .4 = -1.6$

$f(1.4) = f(1.2) + m_{1.2} \Delta x = -1.6 + 1.8(.2) = -1.6 + .36$
$= -1.24$

x	y	m	Δx	Δy	y+Δx
1	−2	2	.2	.4	−2+.4 = −1.6
1.2	−1.6	1.8	.2	.36	−1.6+.36 = −1.24
1.4	<u>−1.24</u>				
1.6					

$\boxed{B}$



6. $\frac{dy}{dx} = 1 + 2y$ $f(0) = 1$
$f(1) = \_\_\_$ $\Delta x = .5$

x	y	m	Δx	Δy	y+Δy
0	1	3	.5	1.5	2.5
.5	2.5	6	.5	3	5.5
1	<u>5.5</u>				

$\boxed{D}$