BC Calculus
Ch. 9 Test Review

Name: ______________________________
Period: _____

AP Practice: Please watch the videos for the FRQ's. They are **very** helpful.

1. 2007 Question 6: https://www.youtube.com/watch?v=yhF85MhaCFU or Search: turksvids 2007 bc calc 6

Let f be the function given by $f(x) = e^{-x^2}$.

(a) Write the first four nonzero terms and the general term of the Taylor series for f about $x = 0$.

(b) Use your answer to part (a) to find $\lim_{x \to 0} \frac{1 - x^2 - f(x)}{x^4}$.

(c) Write the first four nonzero terms of the Taylor series for $\int_0^x e^{-t^2}\,dt$ about $x = 0$. Use the first two terms of your answer to estimate $\int_0^{1/2} e^{-t^2}\,dt$.

(d) Explain why the estimate found in part (c) differs from the actual value of $\int_0^{1/2} e^{-t^2}\,dt$ by less than $\frac{1}{200}$.

2. 2009 Question 6: https://www.youtube.com/watch?v=H4ygQ18ETd0 Search: turksvids 2009 bc calc 6

The Maclaurin series for e^x is $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots + \frac{x^n}{n!} + \cdots$. The continuous function f is defined by $f(x) = \frac{e^{(x-1)^2} - 1}{(x-1)^2}$ for $x \neq 1$ and $f(1) = 1$. The function f has derivatives of all orders at $x = 1$.

(a) Write the first four nonzero terms and the general term of the Taylor series for $e^{(x-1)^2}$ about $x = 1$.

(b) Use the Taylor series found in part (a) to write the first four nonzero terms and the general term of the Taylor series for f about $x = 1$.

(c) Use the ratio test to find the interval of convergence for the Taylor series found in part (b).

(d) Use the Taylor series for f about $x = 1$ to determine whether the graph of f has any points of inflection.

1.	The coefficient of x^6 in the Taylor series expansion about $x = 0$ for $f(x) = \sin\left(x^2\right)$ is (A) $-\frac{1}{6}$ (B) 0 (C) $\frac{1}{120}$ (D) $\frac{1}{6}$ (E) 1
2.	The coefficient of x^3 in the Taylor series for e^{3x} about $x = 0$ is (A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{2}$ (E) $\frac{9}{2}$
3.	A series expansion of $\frac{\sin t}{t}$ is (A) $1 - \frac{t^2}{3!} + \frac{t^4}{5!} - \frac{t^6}{7!} + \cdots$ (B) $\frac{1}{t} - \frac{t}{2!} + \frac{t^3}{4!} - \frac{t^5}{6!} + \cdots$ (C) $1 + \frac{t^2}{3!} + \frac{t^4}{5!} + \frac{t^6}{7!} + \cdots$ (D) $\frac{1}{t} + \frac{t}{2!} + \frac{t^3}{4!} + \frac{t^5}{6!} + \cdots$ (E) $t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \cdots$
4.	What is the value of $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{5^n}$? (A) $-\frac{15}{8}$ (B) $-\frac{9}{8}$ (C) $-\frac{3}{8}$ (D) $\frac{9}{8}$ (E) $\frac{15}{8}$

5. The third-degree Taylor polynomial for a function f about $x = 4$ is $\frac{(x-4)^3}{512} - \frac{(x-4)^2}{64} + \frac{(x-4)}{4} + 2$. What is the value of $f'''(4)$?

(A) $-\frac{1}{64}$ (B) $-\frac{1}{32}$ (C) $\frac{1}{512}$ (D) $\frac{3}{256}$ (E) $\frac{81}{256}$

6. What are all values of x for which the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}\left(x + \frac{3}{2}\right)^n$ converges?

(A) $-\frac{5}{2} < x < -\frac{1}{2}$

(B) $-\frac{5}{2} < x \le -\frac{1}{2}$

(C) $-\frac{5}{2} \le x < -\frac{1}{2}$

(D) $-\frac{1}{2} < x < \frac{1}{2}$

(E) $x \le -\frac{1}{2}$

7. Which of the following is the Maclaurin series for $\frac{1}{(1-x)^2}$?

(A) $1 - x + x^2 - x^3 + \cdots$

(B) $1 - 2x + 3x^2 - 4x^3 + \cdots$

(C) $1 + 2x + 3x^2 + 4x^3 + \cdots$

(D) $1 + x^2 + x^4 + x^6 + \cdots$

(E) $x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots$

8. What is the coefficient of x^6 in the Taylor series for $\frac{e^{3x^2}}{2}$ about $x = 0$?

(A) $\frac{1}{1440}$ (B) $\frac{81}{160}$ (C) $\frac{9}{4}$ (D) $\frac{9}{2}$ (E) $\frac{27}{2}$

9. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{|\sin n|}{n^2}$ II. $\sum_{n=1}^{\infty} e^{-n}$ III. $\sum_{n=1}^{\infty} \frac{n+2}{n^2+n}$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I and III only

10. Consider the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$, where $a_n > 0$ and $b_n > 0$ for $n \ge 1$. If $\sum_{n=1}^{\infty} a_n$ converges, which of the following must be true?

(A) If $a_n \le b_n$, then $\sum_{n=1}^{\infty} b_n$ converges.

(B) If $a_n \le b_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

(C) If $b_n \le a_n$, then $\sum_{n=1}^{\infty} b_n$ converges.

(D) If $b_n \le a_n$, then $\sum_{n=1}^{\infty} b_n$ diverges.

(E) If $b_n \le a_n$, then the behavior of $\sum_{n=1}^{\infty} b_n$ cannot be determined from the information given.

11.	Which of the following statements are true about the series $\sum_{n=2}^{\infty} a_n$, where $a_n = \frac{(-1)^n}{\sqrt{n} + (-1)^n}$? I. The series is alternating. II. $\lvert a_{n+1} \rvert \leq \lvert a_n \rvert$ for all $n \geq 2$ III. $\lim_{n \to \infty} a_n = 0$ (A) None (B) I only (C) I and II only (D) I and III only (E) I, II, and III
12.	The infinite series $\sum_{k=1}^{\infty} a_k$ has nth partial sum $S_n = (-1)^{n+1}$ for $n \geq 1$. What is the sum of the series $\sum_{k=1}^{\infty} a_k$? (A) -1 (B) 0 (C) $\frac{1}{2}$ (D) 1 (E) The series diverges.
13.	What is the sum of the series $\sum_{n=1}^{\infty} \frac{(-2)^n}{e^{n+1}}$? (A) $\frac{-2}{e^2 - 2e}$ (B) $\frac{-2}{e^2 + 2e}$ (C) $\frac{-2}{e+2}$ (D) $\frac{e}{e+2}$ (E) The series diverges.
14.	Let $P(x) = 3 - 3x^2 + 6x^4$ be the fourth-degree Taylor polynomial for the function f about $x = 0$. What is the value of $f^{(4)}(0)$? (A) 0 (B) $\frac{1}{4}$ (C) 6 (D) 24 (E) 144
15.	Which of the following is the Maclaurin series for e^{3x} ? (A) $1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$ (B) $3 + 9x + \frac{27x^2}{2} + \frac{81x^3}{3!} + \frac{243x^4}{4!} + \cdots$ (C) $1 - 3x + \frac{9x^2}{2} - \frac{27x^3}{3!} + \frac{81x^4}{4!} - \cdots$ (D) $1 + 3x + \frac{3x^2}{2} + \frac{3x^3}{3!} + \frac{3x^4}{4!} + \cdots$ (E) $1 + 3x + \frac{9x^2}{2} + \frac{27x^3}{3!} + \frac{81x^4}{4!} + \cdots$
16.	Which of the following series converge? I. $1 + (-1) + 1 + \cdots + (-1)^{n-1} + \cdots$ II. $1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1} + \cdots$ III. $1 + \frac{1}{3} + \frac{1}{3^2} + \cdots + \frac{1}{3^{n-1}} + \cdots$ (A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III
17.	What is the coefficient of x^2 in the Taylor series for $\sin^2 x$ about $x = 0$? (A) -2 (B) -1 (C) 0 (D) 1 (E) 2
18.	The sum of the series $1 + \frac{2^1}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \cdots + \frac{2^n}{n!} + \cdots$ is (A) ln 2 (B) e^2 (C) cos 2 (D) sin 2 (E) nonexistent

19.	Let f be a function with second derivative $f''(x) = \sqrt{1+3x}$. The coefficient of x^3 in the Taylor series for f about $x = 0$ is (A) $\frac{1}{12}$ (B) $\frac{1}{6}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) $\frac{3}{2}$
20.	What is the radius of convergence for the power series $\sum_{n=0}^{\infty} \frac{(x-4)^n}{2 \cdot 3^{n+1}}$? (A) $\frac{1}{3}$ (B) $\frac{3}{2}$ (C) 3 (D) 4 (E) 6
21.	. Let f be a function that has derivatives of all orders for all real numbers, and let $P_3(x)$ be the third-degree Taylor polynomial for f about $x = 0$. The Taylor series for f about $x = 0$ converges at $x = 1$, and $\left\|f^{(n)}(x)\right\| \le \frac{n}{n+1}$ for $1 \le n \le 4$ and all values of x. Of the following, which is the smallest value of k for which the Lagrange error bound guarantees that $\|f(1) - P_3(1)\| \le k$? (A) $\frac{4}{5}$ (C) $\frac{4}{5} \cdot \frac{1}{3!}$ (B) $\frac{4}{5} \cdot \frac{1}{4!}$ (D) $\frac{3}{4} \cdot \frac{1}{4!}$ (E) $\frac{3}{4} \cdot \frac{1}{3!}$
22.	The function f has derivatives of all orders for all real numbers with $f(0) = 3$, $f'(0) = -4$, $f''(0) = 2$, and $f'''(0) = 1$. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$. What is the third-degree Taylor polynomial for g about $x = 0$? (A) $-4x + 2x^2 + \frac{1}{3}x^3$ (C) $3x - 2x^2 + \frac{1}{3}x^3$ (B) $-4x + x^2 + \frac{1}{6}x^3$ (D) $3x - 2x^2 + \frac{2}{3}x^3$ (E) $3 - 4x + x^2 + \frac{1}{6}x^3$
23.	The infinite series $\sum_{k=1}^{\infty} a_k$ has nth partial sum $S_n = \frac{n}{3n+1}$ for $n \ge 1$. What is the sum of the series $\sum_{k=1}^{\infty} a_k$? (A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) 1 (D) $\frac{3}{2}$ (E) The series diverges.
24.	The alternating series test can be used to show convergence of which of the following alternating series? I. $4 - \frac{1}{9} + 1 - \frac{1}{81} + \frac{1}{4} - \frac{1}{729} + \frac{1}{16} - \cdots + a_n + \cdots$, where $a_n = \begin{cases} \frac{8}{2^n} & \text{if } n \text{ is odd} \\ -\frac{1}{3^n} & \text{if } n \text{ is even} \end{cases}$ II. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \cdots + a_n + \cdots$, where $a_n = \frac{(-1)^{n+1}}{n}$ III. $\frac{2}{3} - \frac{3}{5} + \frac{4}{7} - \frac{5}{9} + \frac{6}{11} - \frac{7}{13} + \frac{8}{15} - \cdots + a_n + \cdots$, where $a_n = (-1)^{n+1}\frac{n+1}{2n+1}$ (A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III